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or, as a first approximation,

$$d = 2ga^2/v^2.$$

Remark: If $v = 0$, (1) gives $\dot{\theta} = 0$ or $\theta =$ *constant*, showing that the motion is now uniplanar. Under these conditions (2) reduces to $a\dot{\phi}^2 = 2g \sin \phi$, since $\dot{\phi} \neq 0$. Then $a\ddot{\phi} = g \cos \phi$ or putting $\phi = (\pi/2) + \xi$, $a\ddot{\xi} = - g \sin \xi$ which is the familiar form of the equation of motion of a simple pendulum.

311. Proposed by B. J. BROWN, Student in Drury College.

A particle oscillates in a straight line about a center of force varying as the distance, and is subject to a retardation $k \times (\text{vel.})^2$. If a, b be two successive elongations, on opposite sides, prove that

$$(1 + 2ka)e^{-2ka} = (1 - 2kb)e^{2kb}.$$

What form does the result take if a is infinite?

From Lamb's *Dynamics*, p. 299, ex. 14.

I. SOLUTION BY H. S. UHLER, Yale University.

Let the displacement of the particle at any time t from the center of attraction be denoted by x. To fix the ideas, consider the forces acting on the particle of mass m at an instant when its displacement and velocity are both positive. Then

$$m\frac{d^2x}{dt^2} = - cmx - km \left(\frac{dx}{dt}\right)^2,$$

where c is a positive constant, or

$$\frac{d^2x}{dt^2} + k\left(\frac{dx}{dt}\right)^2 + cx = 0. \tag{1}$$

Equation (1) is a standard form of which the first integral can be obtained at once, for the solution of

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 f(y) + F(y) = 0$$

is

$$\frac{dy}{dx} e^{\int f(y)dy} = \left\{ A - 2\int dyF(y)e^{2\int f(y)dy} \right\}^{\frac{1}{2}}.$$

See Forsyth's *Differential Equations*, third ed., Art. 67.

Hence

$$\frac{dx}{dt} e^{kx} = \left\{ A + \frac{ce^{2kx}}{2k^2}(1 - 2kx) \right\}^{\frac{1}{2}}, \tag{2}$$

The constant of integration A in equation (2) may be found from the datum that $dx/dt = 0$ when $x = - a$, whence

$$\frac{dx}{dt} e^{kx} = \frac{1}{k}\left\{ \frac{c}{2}\left[(1 - 2kx)e^{2kx} - (1 + 2ka)e^{-2ka} \right] \right\}^{\frac{1}{2}}. \tag{3}$$

Again, by hypothesis, $dx/dt = 0$ when $x = b$ so that equation (3) gives the required relation

$$(1 + 2ka)e^{-2ka} = (1 - 2kb)e^{2kb}.$$

When a becomes infinite we have the so-called indeterminate form

$$\frac{1 + 2ka}{e^{2ka}} = 0 + \frac{z}{e^z} = \frac{\infty}{\infty}.$$

In general,

$$\frac{z}{e^z} = \left[z^{-1} + 1 + \frac{z}{2!} + \frac{z^2}{3!} + \cdots + \frac{z^n}{(n + 1)!} + \cdots \right]^{-1};$$

consequently as z increases indefinitely ze^{-z} approaches zero as a limit. The same result follows from the well-known fact that "any infinite number is an infinity of higher order than any power of its logarithm."

Finally, since b is not negative, $e^{2kb} \neq 0$ and

$$b = \frac{1}{2k}.$$

Remark: If k were zero the last result would become $b = \infty$ which is consistent with the fact that equation (1) now represents rectilinear simple harmonic motion with infinite amplitude $a = b = \infty$.

II. Solution by Horace Olson, Chicago, Illinois.

From the conditions of the problem, $dv/dt = -cx \mp kv^2$, if the center of force is at $x = 0$. The upper or lower sign of kv^2 is taken according as the particle is moving in a positive or a negative direction.

From the identity $dv/dt = dv/dx \cdot dx/dt = v\,dv/dx$, we have $v\,dv/dx = -cx \mp kv^2$. Set $w = v^2$. Then $dw/dx = -2cx \mp 2kw$, a linear differential equation. Whence,

$$we^{\pm 2kx} = \mp \frac{cxe^{\pm 2kx}}{k} + \frac{ce^{\pm 2kx}}{2k^2} + C.$$

v, and, therefore, $w = 0$ when $x = a$ or b. Hence,

$$0 = \left(\frac{ac}{k} + \frac{c}{2k^2}\right)e^{-2ka} + C = \left(-\frac{bc}{k} + \frac{c}{2k^2}\right)e^{2kb} + C;$$

whence

$$(1 + 2ka)e^{-2ka} = (1 - 2kb)e^{2kb}.$$

When $a = +\infty$, the first member of this equation becomes 0 and $(1 - 2kb)e^{2kb} = 0$. Hence, if b is not $-\infty$, $2kb = 1$.

When $a = -\infty$, we have $-\infty = (1 - 2kb)e^{2kb}$; whence $b = +\infty$.

QUESTIONS AND DISCUSSIONS.

Send all communications to U. G. Mitchell, University of Kansas, Lawrence, Kansas.

NEW QUESTIONS.

32. In a discussion of the Peaucellier[1] Cell by analytic methods the following equations are obtained:

$$(1)\ (x_2 - x_1)^2 + (y_2 - y_1)^2 - b^2 = 0; \qquad (2)\ (x_3 - x_1)^2 + (y_3 - y_1)^2 - b^2 = 0;$$

$$(3)\ (x_2 - X)^2 + (y_2 - Y)^2 - b^2 = 0; \qquad (4)\ (x_3 - X)^2 + (y_3 - Y)^2 - b^2 = 0;$$

$$(5)\quad x_2^2 + y_2^2 - K^2 = 0; \qquad (6)\quad x_3^2 + y_3^2 - K^2 = 0;$$

$$(7)\ x_1^2 + y_1^2 - 2cx_1 = 0.$$

The result of eliminating x_1, y_1, x_2, y_2, x_3, y_3 gives an equation of the first degree, which establishes that the linkage will trace a straight line. There are various ways of effecting this elimination.

I. What element of the situation is left unused by the following procedure in the elimination?

(*a*) From equations (1), (3), (5) eliminate x_2 and y_2 and obtain an equation

$$f_1(x_1, y_1) = 0. \tag{8}$$

(*b*) From equations (2), (4), (6) eliminate x_3 and y_3, and obtain an equation

$$f_2(x_1, y_1) = 0. \tag{9}$$

(*c*) From equations (7), (8), (9) eliminate x_1 and y_1, and obtain the desired equation.

II. How should this procedure be supplemented to secure the result?

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[1] If reference is made to the article on "Linkages" in the December (1915) Monthly by Mr. Leavens, the following coördinates may be applied to his figure: $O(o, o)$; $C(c, o)$; $P_1(x_1, y_1)$; $M(x_2, y_2)$; $M_1(x_3, y_3)$; $P_2(XY)$.